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Heat Transfer on Power Law Bodies

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Nomenclature

δ = boundary-layer thickness
τ = shear stress
r = radius
P = pressure
ρ = density
u = velocity parallel to surface
μ = viscosity
X = coordinate distance measured along body
y = coordinate distance measured normal to body
M = Mach number
γ = ratio of specific heats
F = a constant
n = exponent of power body
l = Prandtl mixing length
D = e^{-FK}
a = $K_{U\infty}(\tau_\omega/\rho_\omega)^{1/2}$
α = $(2A^2 - B)/(B^2 + 4A^2)^{1/2}$
β = $B/(B^2 + 4A^2)^{1/2}$
A = $[(\gamma - 1)/2]M_\infty/(\tau_\omega/\tau_\infty)$
B = $(\{1 + [(\gamma - 1)/2]M_\infty{}^2\}/(\tau_\omega/\tau_\infty)) - 1$
K = l/y

Subscripts

ω = wall
∞ = edge of boundary layer or freestream
A = axially symmetric
$2D$ = two-dimensional

Introduction

The development of an expression for the ratio of two-dimensional to axially symmetric shear stress in laminar flow for bodies of the form $r = cx^n$ was shown in Ref. 1; the result was

$$(\tau_A/\tau_{2D}) = (1 + 2n)^{1/2}$$

Fully developed turbulent flow is considered in the current note; an expression similar to that for laminar flow is derived:

$$(\tau_A/\tau_{2D}) = (1 + n)^{1/5}$$

Discussion

The von Kármán momentum equation

$$\frac{\partial}{\partial X}\int_0^\delta r\rho u\,(u_\infty - u)dy = -\frac{\partial P}{\partial X}\int_0^\delta r dy - r_\omega\tau_\omega \quad (1)$$

can be reduced to

$$\frac{\partial}{\partial X}\int_0^\delta \rho u(u_\infty - u)dy + \frac{1}{r_\omega}\frac{\partial_{r_\omega}}{\partial X}\int_0^\delta \rho u\,(u_\infty - u)dy = -\tau_\omega \quad (2)$$

by assuming that the boundary-layer thickness is thin compared to the body dimension ($\delta \ll r$) and that $\partial P/\partial X = 0$. Reference 2 shows that for a flat plate [where $(1/r_\omega)(\partial_{r_\omega}/\partial X) = 0$]

$$\frac{\rho_\omega u_\infty}{\mu_\omega} = \frac{D}{K^3(1 + B - A^2)^{1/2}}\,a^2\,\frac{d}{dX}\times \left\{\exp\left[\frac{a}{A}\,(\sin^{-1}\alpha + \sin^{-1}\beta)\right]\right\} \quad (3)$$

Equation (3) was integrated to

$$\frac{\rho_\omega u_\infty X}{\mu_\omega} = \frac{D}{K^3(1 + B - A^2)^{1/2}}\,a^2 \times \exp\left[\frac{a}{A}\,(\sin^{-1}\alpha + \sin^{-1}\beta)\right] \quad (4)$$

The introduction of $1/X = (1/r\omega)(\partial r\omega/\partial X)$ (for cones) in Eq (2) produced a new term in (3):

$$\frac{\rho_\omega u_\infty}{\mu_\omega} = \frac{D}{K^3(1 + B - A^2)^{1/2}}\times \left(\frac{d}{dX}\left\{a^2\exp\left[\frac{a}{A}\,(\sin^{-1}\alpha + \sin^{-1}\beta)\right]\right\} + \frac{1}{X}\left\{a^2\exp\left[\frac{\alpha}{A}\,(\sin^{-1}\alpha + \sin^{-1}\beta)\right]\right\}\right) \quad (5)$$

Equation (5) was integrated to

$$\frac{1}{2}\,\frac{\rho_\omega u_\infty X}{\mu_\omega} = \frac{D}{K^3(1 + B - A^2)^{1/2}}\,a^2 \times \exp\left[\frac{a}{A}\,(\sin^{-1}\alpha + \sin^{-1}\beta)\right] \quad (6)$$

The current effort is an extension to Ref. 2 for more general body shapes described by the equation $r = cx^n$; $(1/r_\omega)\cdot(\partial_{r_\omega}/\partial X) = n/X$ can be substituted into Eq. (2), thereby changing Eq. (5) to

$$\frac{\rho_\omega u_\infty}{\mu_\omega} = \frac{D}{K^3(1 + B - A^2)^{1/2}}\times \left(\frac{d}{dX}\left\{a^2\exp\left[\frac{a}{A}\,(\sin^{-1}\alpha + \sin^{-1}\beta)\right]\right\} + \frac{n}{X}\left\{a^2\exp\left[\frac{a}{A}\,(\sin^{-1}\alpha + \sin^{-1}\beta)\right]\right\}\right) \quad (7)$$

This is a differential equation of the form

$$\Delta' + (n/X)\Delta = \Lambda \quad (8)$$

where

$$\Delta' = \frac{d}{dX}\left\{a^2\exp\left[\frac{a}{A}\,(\sin^{-1}\alpha + \sin^{-1}\beta)\right]\right\}$$

$$\Delta = \left\{a^2\exp\left[\frac{a}{A}\,(\sin^{-1}\alpha + \sin^{-1}\beta)\right]\right\}$$

$$\Lambda = \frac{\rho_\omega u_\infty}{\mu_\omega}\,\frac{K^3(1 + B - A^2)^{1/2}}{D}$$

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By use of the integrating factor $\exp\int(n/X)dX$, Eq. (8) is integrated to

$$\Delta = [\Lambda X/(n+1)] + C \tag{9}$$

Since $C = 0$ at $X = 0$, Eq. (9) can be expressed as

$$\frac{1}{1+n}\cdot\frac{\rho_\omega u_\infty X}{\mu_\omega} = \frac{D}{K^3(1+B-A^2)^{1/2}} \times \left\{a^2 \exp\left[\frac{a}{A}(\sin^{-1}\alpha + \sin^{-1}\beta)\right]\right\} \tag{10}$$

Comparison of Eq. (10) with Eq. (4) shows that the axially symmetric heat transfer is the same as the two-dimensional value when the Reynolds number is divided by $(1 + n)$. Note that for $n = 1$ the preceding reduces to Eq. (6), which agrees exactly with Eq. (23) of Ref. 2. Finally, since the turbulent shear stress varies inversely with the $\frac{1}{5}$ power of the Reynolds number, then

$$(\tau_A/\tau_{2D}) = (1+n)^{1/5} \tag{11}$$

The assumptions that $\delta \ll r$ and $\partial P/\partial X = 0$ are valid only at $X = \infty$. However, Ref. 1 shows that for an expression similar to Eq. (11) (which was also developed at $X = \infty$) the results are acceptable at smaller values of X. This fact tends to indicate (although no proof is available) that the turbulent results shown here will give similar accuracy at the more practical values of X.

Thus, a simple relationship has been developed to transform two-dimensional shear stress to axially symmetric shear stress for bodies of the form $r = cx^n$.

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Backside Temperatures of an Internal Insulator in a Solid-Propellant Motor

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Nomenclature

R = average erosion rate
t_i = initial temperature of insulator
t_d = decomposition temperature of insulator
t_b = backside temperature of insulator
x_i = initial insulator thickness
u = insulator thickness at time θ
θ = exposure time of insulator surface
α = thermal diffusivity of insulator

SOLID-PROPELLANT rocket motors are internally insulated to minimize strength degradation of the pressurized case during propellant burning time. The solid propellant is an effective self-insulator and helps to protect the motor case from the chamber environment until the flame front reaches the internal insulator surface.[1] The exposure time of the internal insulator surface to the hot and erosive chamber gases can be predicted from a knowledge of the propellant design and burning rate. A simple end-burning propellant design allows the insulator surface to become gradually exposed, whereas internal-burning designs, such as a wagon wheel or star, result in almost the entire insulator surface being exposed simultaneously at the end of the propellant burning time. Usually, a more complex propellant design is required, and the exposure time of the insulator surface varies circumferentially around the motor case and longitudinally along the motor case. The internal insulator will undergo erosion, which can be measured in a post-firing inspection. An average erosion rate for any point on the insulator may be calculated from the measured erosion and exposure time, or the erosion rate may be estimated from prior data. This technical note reports an investigation in which the average erosion rate of a silica-filled Buna-N internal insulator and exposure time of the insulator surface to the chamber environment were related to estimate the backside temperature of the insulator. A comparison was made between predicted and experimental insulator backside temperatures for various initial thicknesses in a full-scale, modified, double-base, solid-propellant rocket motor. The experimental temperature data were obtained by using no. 28 American wire gage iron-constantan thermocouples.

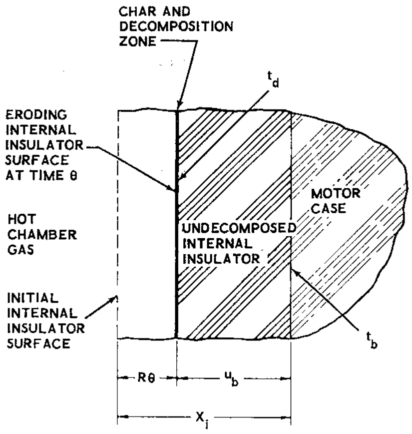

Fig. 1 Diagram of eroding insulator at time θ.

Literature contains many articles on heat conduction with a moving boundary or heat source. Eckert,[2] Schneider,[3] and Carslaw and Jaeger[4] treat various cases. This investigation is based on the assumption that the internal insulator erodes at a constant average rate R at a particular point on the insulator during exposure to hot chamber gases. A large temperature gradient exists through the surface film, insulator char, and decomposition zones. Thin char and decomposition zones are assumed to have a constant thickness. Under these conditions, temperature of the cool side of the decomposition zone, t_d, is independent of chamber environment, temperature, pressure, velocity, etc., which affect the rate of erosion. The insulator backside temperature is t_b.

A moving coordinate system is used with the origin at the exposed surface of the insulator. With this system, the backside surface is moving at the constant rate R toward the insulator surface. A quasi-steady state exists with $\partial t/\partial\theta = 0$ at any point in the material referred to the moving coordinate system. This system is illustrated in Fig. 1. It is assumed that thermal properties of the undecomposed insulator are constant and that heat flow is unidirectional and follows the unsteady-state conduction law

$$\partial t/\partial\theta = \alpha(\partial^2 t/\partial x^2) \tag{1}$$

With a boundary moving at a constant rate, u is related to x and θ by

$$u = x - R\theta \tag{2}$$

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